On the polarization state of X-rays generated using a rotating four-quadrant X-ray phase retarder system

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1. Introduction

The transmission-type X-ray phase retarder (Hirano et al., 1991; Giles, Malgrange, Goulon, de Bergevin, Vettier, Darttye et al., 1994; Giles, Malgrange, Goulon, de Bergevin, Vettier, Fontaine et al., 1994; Hirano et al., 1992, 1993, 1995) opened up a new feasability to control the polarization state of synchrotron radiation. However, in the optical device it had problems of phase-shift inhomogeneity owing to the finite angular divergence and energy spread of the incident X-rays, i.e. off-axis and chromatic aberrations. The present author and his coauthors developed two-quadrant and four-quadrant phase retarder systems that consist of two and four transmission-type phase retarders, respectively. The former can compensate for off-axis aberration (Okitsu et al., 2001) and the latter can compensate for both off-axis and chromatic aberrations (Okitsu et al., 2002).

Further, the present author and coauthors developed a ‘rotating four-quadrant phase retarder system’ that can generate arbitrarily polarized X-rays from the horizontally polarized synchrotron X-rays. They also recorded six-beam pinhole topographs for a parallel-plate silicon crystal where quantitative in good agreement with computer-simulated pinhole topographs for a parallel-plate silicon crystal. Details and results of this experiment are given in a separate paper [Okitsu et al. (2011), Acta Cryst. A67, 550–556].

2. Correction for the polarization states of X-rays generated by the phase retarder system

The change of polarization state can be described using Jones calculus (Jones, 1941) in general. The amplitude vector whose original polarization state is horizontal is represented by a Jones vector, $D_0 = (D_{0L}^{(0)}, D_{0R}^{(0)}) = (1, 0)$. Here, $D_{0L}^{(0)}$ and $D_{0R}^{(0)}$ are the complex amplitudes of X-rays for which the horizontal and vertical, respectively. $D_0$ is transformed to $D'_0$ by transmission through the four-quadrant phase retarder system (Okitsu et al., 2002) as follows,

$$D'_0 = R \Phi R^{-1} D_0,$$

where

$$R = \begin{bmatrix} \cos(45^\circ + x_{\text{PR}}) & -\sin(45^\circ + x_{\text{PR}}) \\
\sin(45^\circ + x_{\text{PR}}) & \cos(45^\circ + x_{\text{PR}}) \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \exp(-i\Delta \phi_{\text{total}}/2) & 0 \\
0 & \exp(i\Delta \phi_{\text{total}}/2) \end{bmatrix}.$$  

Here, $\Delta \phi_{\text{PR}}$ is the rotation angle of the phase retarder system and $\Delta \phi_{\text{total}}$ is the total phase shift given by the phase retarder system, as defined by O et al. 2006. Hereafter, amplitude vectors are column vectors in the matrix calculations whereas they are described as raw vectors in the text.

On the other hand, $D'_0$ can necessarily be written as a linear combination of left- and right-screwed circular polarizations whose complex amplitude vector $D_{0 \text{LR}} = (D_{0L}^{(0)}, D_{0R}^{(0)})$ as follows,

$$D'_0 = \frac{D_{0L}^{(0)}}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} + \frac{D_{0R}^{(0)}}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}.$$  

Therefore,

$$D'_0 = M_c D_{0 \text{LR}},$$

where

$$M_c = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}.$$
By substituting (1) into (4), \( \mathbf{D}_0^{(LR)} \) can be obtained by
\[
\mathbf{D}_0^{(LR)} = \mathbf{M}_2 \mathbf{R} \mathbf{R}^\dagger \mathbf{D}_0.
\] (5)

Because the lengths of the major and minor axes of the elliptical polarization \( \mathbf{D}_0^{(LR)} \) are \( 2(|\mathbf{D}_0^{(L)}| + |\mathbf{D}_0^{(R)}|) \) and \( 2(|\mathbf{D}_0^{(L)}| - |\mathbf{D}_0^{(R)}|) \), the ellipticity \( R \) is given by
\[
R = \frac{|\mathbf{D}_0^{(L)}| - |\mathbf{D}_0^{(R)}|}{|\mathbf{D}_0^{(L)}| + |\mathbf{D}_0^{(R)}|},
\] (6)

where the sign of \( R \) is positive when the polarization is left-screwed. Further, the directions of the electric field vectors of left- and right-handed circular polarizations described as \( \mathbf{D}_0^{(L)}(1, -i)/\sqrt{2} \) and \( \mathbf{D}_0^{(R)}(1, i)/\sqrt{2} \) are inclined by \( \arg(\mathbf{D}_0^{(L)}) \) and \( -\arg(\mathbf{D}_0^{(R)}) \), respectively, viewed from the downstream direction when \( \mathbf{v} - \mathbf{K}_0 \cdot \mathbf{r} = 0 \), because the rotation helicities are contrary between the left- and right-handed circular polarizations. Here, the amplitudes of X-rays with CL polarizations were generated. In the experiment of O et al. (2011), obtained from the experimental result shown in Fig. 6 of O et al. (1995), the phase retarder crystal, X-rays, energy. 2011, obtained from the experimental result shown in Fig. 6 of O et al. (2011) were used for generating the phase retarder crystal, X-rays, energy. 2011, obtained from the experimental result shown in Fig. 6 of O et al. (2011) were used for generating the phase retarder crystal, X-rays, energy. 2011, obtained from the experimental result shown in Fig. 6 of O et al. (2011).

The present work was performed at the High-Power X-ray Laboratory, Nano-Engineering Research Center, Institute of Engineering Innovation, Graduate School of Engineering, The University of Tokyo, Japan. Practical calculations of polarization state of X-rays based on the present work were performed using the facilities of the Supercomputer Center, Institute for Solid State Physics, The University of Tokyo, Japan. The present work is one of the activities of the Active Nano-Characterization and Technology Project financially supported by the Special Coordination Fund of the Ministry of Education, Culture, Sports, Science and Technology of the Japanese government.

References


